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The peculiarities of discrete diffraction in two-dimensional optical waveguide arrays

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Abstract

The fundamental peculiarities of discrete diffraction in the two-dimensional waveguide arrays were studied. The discrete diffraction properties of such arrays can be effectively altered, depending upon the input conditions. By slightly changing the input conditions, light can experience normal diffraction in one-direction and experience anomalous diffraction in the other.

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Discrete waveguide arrays have become a topic of considerable attention for many years. These periodic structures exhibit a wealth of phenomena that have no analog in the continuous regime [1–4]. For example, due to the periodic nature of waveguide arrays, their diffraction behavior can be tailored depending on the propagation k-vector within the Brillouin zone [3,4]. As a result, zero or even reverse diffraction is possible in these waveguide array structures. Discrete solitons are also known to exist in such nonlinear array systems through the interplay of linear coupling effects and material nonlinearity. Many properties of optical discrete spatial solitons have been systematically explored in theory and experiment, including generalizations to diffraction management [3,5], and diffraction-managed solitons [6].

The above mentioned studies have been limited primarily to the one-dimensional waveguide arrays, but interest has been stimulated in two-dimensional waveguide arrays since the first experimental observation of discrete solitons was achieved in one-dimensional AlGaAs waveguide arrays [2]. This experimental observation stimulated much new research, such as studies of solitons in two-dimensional photorefractive optical lattices in which localization phenomena have been observed [7–9]. But the optically induced lattices in photorefractives are limited to periodic configurations and are not permanent. To overcome these disadvantages, other methods to create two-dimensional arrays are also being developed. These include arrays of fiber bundles [10], and optically-written arrays in silica [11]. This optically-written technique has provided highly uniform linear arrays and holds much potential for the observation of nonlinear effects as well [12]. It was demonstrated that one can write optical waveguides along arbitrary paths by tightly focusing ultrashort laser pulses into silica glass .With this technique the field energy is deposited in the focal volume and a permanent refractive-index increase is induced. Moving the sample with respect to the focus of the beam can create low-loss optical waveguides and can fabricate two-dimensional waveguide arrays with designed diffraction. In this letter, stimulated by the idea of diffraction management in one-dimensional waveguide arrays, we study the properties of discrete diffraction in two-dimensional waveguide arrays environment and we find that the diffraction behavior can be drastically altered depending on the propagation k-vector within the Brillouin zone. As will be shown, these structures possess significantly more complex than their 1D counterpart.

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We begin with tracing diffraction in the homogenous medium, we consider the propagation of scalar waves of the form $E(r) = E_0 \exp(i\bar{k} \cdot \bar{r})$ in a three-dimensional free space, where $\bar{k}$ is the wave vector whose x, y and z components are k_x, k_y and k_z, respectively. The absolute value of $\bar{k}$ is $k = 2\pi n/\lambda_0$, and they are related by the condition $k^2 = k_x^2 + k_y^2 + k_z^2$ or equivalently, $k_z = \sqrt{k^2 - k_x^2 - k_y^2}$. When an optical field propagates over a distance L, each transverse component of the frequency k_x, k_y gains a phase $k_z(k_x, k_y)L$, the initial profile of the beam along the x-, y-directions broadens as the beam propagates due to the phase accumulated by the different spatial frequencies. In analogy with temporal dispersion, the spatial broadening of the beam is related to $D_x = \partial^2 k_z/\partial k_x^2$, $D_y = \partial^2 k_z/\partial k_y^2$ in the x-, y-directions respectively. In homogeneous media, D_x and D_y are always negative, implying that diffraction in such media is equivalent to anomalous dispersion.

Let us now analyze the spatial broadening of a beam in the discrete 2D waveguide arrays, considering two-dimensional waveguide arrays are lossless and infinite and that they are comprised of identical, regularly spaced waveguides. The distance between successive waveguides is d. Fig. 1(a) depicts such two-dimensional waveguide structure. By using the formalism of coupled-mode theory and by considering only the around waveguides coupling, it can be shown that the electric field propagating in the (n,m)th waveguide obeys the following equation

$$\begin{aligned}\frac{dE_{n,m}}{dz} = {} & i\beta E_{n,m} + ic_1(E_{n+1,m} + E_{n-1,m} + E_{n,m-1} + E_{n,m+1}) \\ & + ic_2(E_{n+1,m+1} + E_{n+1,m-1} + E_{n-1,m-1} + E_{n-1,m+1}) \\ & + i\gamma |E_{n,m}|^2 E_{n,m} \end{aligned} \quad (1)$$

where β is the field propagation constant of each waveguide, c_1 and c_2 are the coupling coefficients of nearest-neighbor and next-nearest-neighbor, which are proportional to an overlap integral of the two modes of such waveguides. Fig. 1(b) is the sketch drawing of coupling due to field overlap of such waveguides. The last term describes the nonlinear Kerr effect, with a coefficient γ. The nonlinear term is significant only at high powers and can be ignored in the low-intensity regime. This low-power linear solution describes discrete diffraction in the 2D array of waveguides. When a single, or few, input guides are excited with low optical power, light spreads over more and more waveguides as it propagates through discrete diffraction.

The linear spatial dispersive relation of two-dimensional waveguide arrays can be obtained from Eq. (1) by assigning to $E_{n,m}$ the form $E_{n,m} = A\exp[i(k_z z + k_x x_n + k_y y_m)]$, where $x_n = nd$, $y_m = md$. In this case, the linear dispersion equation readily follows and is given by

$$k_z = \beta + 2c_1(\cos k_x d + \cos k_y d) + 4c_2 \cos k_x d \cos k_y d \quad (2)$$

The dispersive character of the two-dimensional waveguide arrays are evident when only one waveguide is initially excited, for example, $E_{0,0} = A_0$ and $E_{n,m} = 0$ for $(n \neq 0, m \neq 0)$ at $z = 0$. In order to study the properties of discrete diffraction, we define the discrete diffraction coefficients in the x-, y-directions respectively as follow

$$D_x = \frac{\partial^2 k_z}{\partial k_x^2} = -2c_1 d^2 \cos(k_x d) - 4c_2 d^2 \cos(k_y d)\cos(k_x d) \quad (3)$$

$$D_y = \frac{\partial^2 k_z}{\partial k_y^2} = -2c_1 d^2 \cos(k_y d) - 4c_2 d^2 \cos(k_x d)\cos(k_y d) \quad (4)$$

A Brillouin zone is formed in the range $|k_x d| < \pi$ and $|k_y d| < \pi$, any higher frequency has an equivalent inside it. Especially, when we fix $c_1 = 0.284\ \text{mm}^{-1}$, $c_2 = 0.043\ \text{mm}^{-1}$ [13], from the Eqs. (3) and (4), we can get the domain of the sign of the diffraction, as it is depicted in Fig. 2. D_x and D_y become positive in the range $\pi/2 < |k_x d| \leqslant \pi$ and $\pi/2 < |k_y d| \leqslant \pi$ respectively, enabling light beams to experience anomalous diffraction, i.e, of opposite sign to that experienced in nature. In practice, the sign and value of the diffraction can be controlled

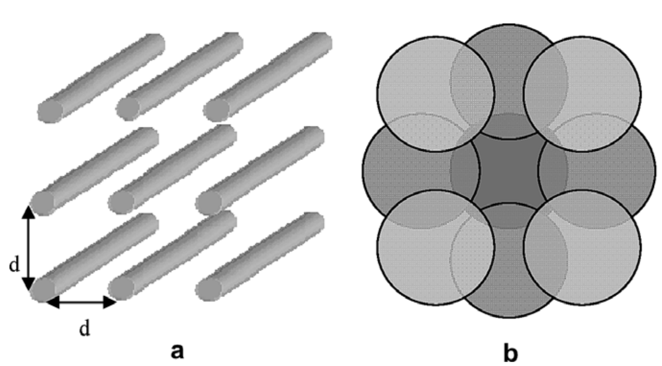

Fig. 1. (a) Schematic drawing of the two-dimensional waveguides arrays. (b) The sketch drawing of coupling due to field overlap of each waveguide in 2D waveguide arrays.

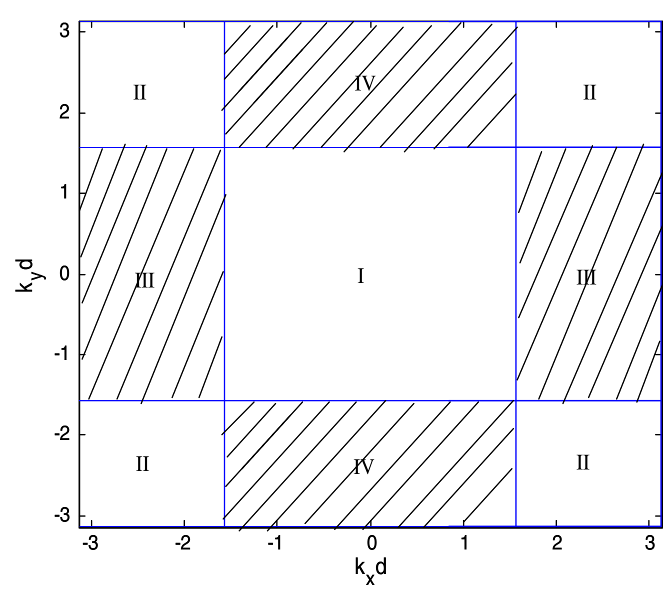

Fig. 2. The sign of the diffraction coefficients D_x and D_y in different ranges. (I): $D_x < 0$ and $D_y < 0$. (II): $D_x > 0$ and $D_y > 0$. (III): $D_x > 0$ and $D_y < 0$. (IV): $D_x < 0$ and $D_y > 0$

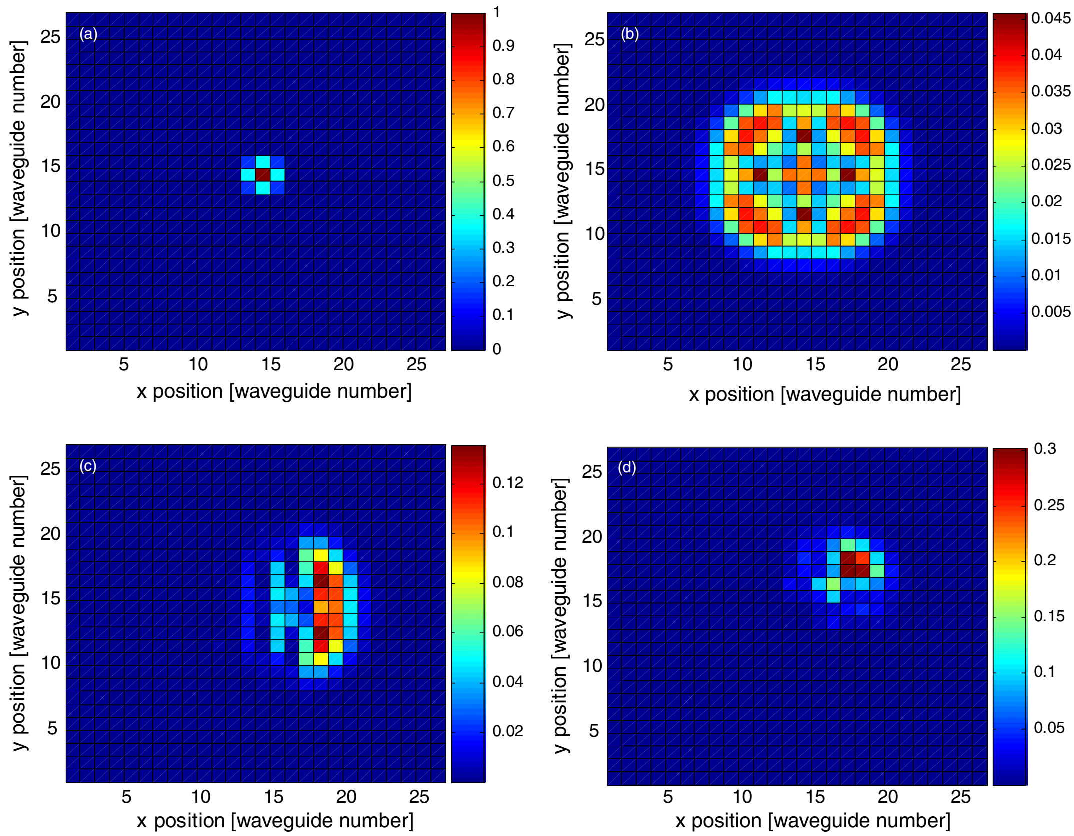

Fig. 3. Spatial intensity distribution for 2D waveguide arrays, when light is injected into the centre, with the maximal field magnitude $E = 1$; (a) spatial intensity distribution of the initial injected beam at $z = 0$ mm; (b) spatial intensity distribution for $k_xd = 0$, $k_yd = 0$ at $z = 8$ mm; (c) spatial intensity distribution for $k_xd = \pi/2$, $k_yd = 0$ at $z = 8$ mm; (d) spatial intensity distribution for $k_xd = \pi/2$, $k_y\ d = \pi/2$ at $z = 8$ mm.

and manipulated by launching light at a particular angle or equivalently by tilting the waveguide arrays. This in turn allows the possibility of achieving a self-defocusing (with positive Kerr coefficient) regime which leads to the formation of discrete dark solitons [4]. The tilted angle α and γ are related to the wave numbers k_x, k_y by the relations [3] $\sin\alpha = k_x/k$, $\sin\gamma = k_y/k$ (α, γ are the tilted angles of the input beam in the x-, y-directions respectively). The angles corresponded to various values $\theta = k_xd$ (or $\theta = k_yd$) in the range of 0–π, if the input beam at a wavelength of 1.53 μm and the linear refractive index $n = 1.5$, $d = 16$ μm, then π equivalent to a tilt angle of $\alpha = \sin^{-1}(\theta/kd) \approx 1.8°$ (or $\gamma = \sin^{-1}(\theta/kd) \approx 1.8°$) under these condition. Moreover we can see that D_x and D_y completely disappears around these points $k_x = \pm\frac{\pi}{2d}$ and $k_y = \pm\frac{\pi}{2d}$ respectively. Clearly, the sign and value of diffraction are determined by three physical parameters, namely the period, the coupling strength, and the tilted angle.

It is also fundamental to understand the diffractive properties of these 2D discrete systems when broader beams, exciting more than one waveguide element, are involved. Under these circumstances the modal fields within waveguide n, m are written in the form $E_{n,m} = u_{n,m}\exp[\mathrm{i}(k_zz + k_xx_n + k_yy_m)]$, Substituting this form into Eq. (1) we obtain

$$\begin{aligned}
&\mathrm{i}c_1[u_{n+1,m}\exp(\mathrm{i}k_xd) + u_{n-1,m}\exp(-\mathrm{i}k_xd) + u_{n,m-1}\exp(-\mathrm{i}k_yd)\\
&\quad + u_{n,m+1}\exp(\mathrm{i}k_yd)] + \mathrm{i}c_2[u_{n+1,m+1}\exp(\mathrm{i}k_xd + \mathrm{i}k_yd)\\
&\quad + u_{n+1,m-1}\exp(\mathrm{i}k_xd - \mathrm{i}k_yd) + u_{n-1,m-1}\exp(-\mathrm{i}k_xd - \mathrm{i}k_yd)\\
&\quad + u_{n-1,m+1}\exp(-\mathrm{i}k_xd + \mathrm{i}k_yd)] - \mathrm{i}[2c_1\cos(k_xd)\\
&\quad + 2c_1\cos(k_yd) + 4c_2\cos(k_xd)\cos(k_yd)]u_{n,m}\\
&\quad + \mathrm{i}\gamma|u_{n,m}|^2u_{n,m} = \frac{\mathrm{d}u_{n,m}}{\mathrm{d}z}
\end{aligned} \tag{5}$$

Notice that Eq. (5) is relation with k_xd and k_yd, thus, in the linear regime, the diffraction behavior of the array can be tailored or altered, depending on the values of k_xd and k_yd. It can be normal diffraction in one-direction and anomalous diffraction in the other, depending on the tilted angle. We carried out the simulations by exactly solving Eq. (5) under linear conditions. Assuming that the 2D waveguide arrays have the 27×27 structure, and the length of each waveguide is 8 mm long, the input Gaussian beam with $w_0 = 38$ μm excited mostly nine waveguides. Fig. 3 shows the peculiarities of discrete diffraction when a 2D circular Gaussian beam is launched into the waveguide with different values of k_xd and k_yd corresponding to different tilted angles. When the beam is normal injected into the waveguide arrays with low optical power, light spreads over more and more waveguides as it propagates through discrete diffraction, and the spatial intensity distribution is central symmetry (see Fig. 3(b)). For $k_x d = \pi/2$ and $k_y d = 0$, where diffraction should vanish in the x-direction and be the normal diffraction in the y-direction, we can see from Fig. 3(c) that the output field along y-axis broadens much larger than x-axis, it also exhibits an asymmetric profile along x-direction, attributed to the remanent third-order diffraction related to x. These results demonstrate that the properties of discrete diffraction in the two-dimensional waveguide arrays are more complex than their 1D counterpart.

In conclusion, we have studied the discrete diffraction behavior of two-dimensional waveguide arrays in theory. Light can experience normal diffraction in one-direction and experience anomalous diffraction in the other in these waveguide array structures by slightly changing the input conditions, for example, at different tilted angles.

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